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When are Superconductors Really Superconducting?

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While the most striking aspect of superconductivity is dissipation-free current flow, it is not straightforward to experimentally demonstrate whether the resistance is truly zero or "merely" immeasurably small. The distinction between zero or very small resistance is important, because the true superconducting state can be destroyed by thermodynamic fluctuations. This paper discusses a variety of different superconducting systems: two-dimensional superconductors, Josephson-junction arrays, and three-dimensional superconductors in zero and nonzero magnetic field, and the experiments conducted to determine which, and under what conditions, systems are *really* superconducting.

KEY WORDS: superconductivity; phase transitions; Kosterlitz-Thouless transition; vortex-glass transition.

1. INTRODUCTION

The hallmark of superconductivity, and the basis of its name, is zero resistance for temperature T below a transition temperature T_c. Since the discovery of superconductivity, much research has been done to understand the limits of this zero resistance state [1]. It is known, for example, that a magnetic field greater than the critical field $H_c(T)$ will destroy superconductivity in a type-I superconductor, as will a current density greater than the critical current density $J_c(T)$.

One other property that determines whether the resistance goes to zero, or is, perhaps, just very small, is the spatial dimensionality of the system, D. In zero magnetic field, and for small currents, we know that the resistance is not strictly zero for one-dimensional systems. The resistance is zero for $D = 3$, and for $D = 2$, some systems do not have zero resistance, while others will. Interestingly, the two-dimensional superconductors that have zero resistance also have zero critical current.

Roughly 50 years ago, two remarkable theories provided us with an understanding of superconductivity. The phenomenological Ginzburg-Landau theory assumed a superconducting order parameter analogous to the order parameter in other second-order phase transitions. The Ginzburg-Landau theory has been very successful in describing the behavior of superconductors on the macroscopic level. The microscopic Bardeen-Cooper-Schrieffer theory showed that a weak attractive electron–electron interaction leads to a superconducting state. Both theories, however, make mean-field assumptions [1]. Because of this, thermal fluctuations are not taken into account, and fluctuations make the existence of zero resistance a subtle and interesting question.

At any nonzero temperature, fluctuations occur because a system can borrow an energy kT from its environment. For $T < T_c$, this makes it possible to temporarily increase the energy of a small volume of superconductor, perhaps enough to drive the small volume into the normal state. The effects of such fluctuations may be relatively benign, perhaps weakening superconductivity without destroying it. Under the right circumstances, however, fluctuations can destroy the superconducting state.

There are two key physical quantities, the correlation length ξ, and the correlation time τ, that together characterize the fluctuations [2]. In three dimensions, the correlation length diverges at T_c, varying as

$$\xi \sim \varepsilon^{-\nu} \tag{1}$$

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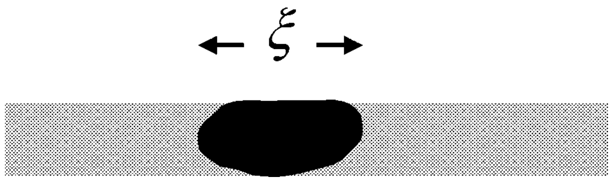

Fig. 1. Schematic picture of a superconducting wire, shown in gray. The fluctuation (shown in black) is large enough to cross the entire wire, making the system essentially one dimensional and destroying the superconductivity.

where $\varepsilon = |T - T_c|/T_c$ and ξ is the size of a typical fluctuation. Above T_c, ξ is the size of superconducting regions that occur as fluctuations in the normal background, and below T_c, ξ is the size of normal fluctuations in the superconducting background. The correlation time varies as

$$\tau \sim \xi^z \sim \varepsilon^{-z\nu} \tag{2}$$

and is a measure of the lifetime of fluctuations. Eqs. (1) and (2) contain the static and dynamic critical exponents ν and z which characterize the diverging length and time scales. Note that for the special case of $D = 2$, the correlation length diverges, but not as a power of ε. For $D = 2$, however, τ will still vary as ξ^z.

It is instructive to briefly consider the case where $D = 1$. In order to be in the one-dimensional limit, a superconducting wire must be close enough to T_c for the correlation length, $\xi(T)$, to be larger than the diameter of the wire. In this limit, at nonzero temperature there will be fluctuations that are large enough to drive the entire cross section of the sample normal, as shown schematically in Fig. 1. This will cause a nonzero resistance, and will destroy the long-range coherence in the sample. Since the probability of such a fluctuation is given by a Boltzmann factor, it is nonzero (although perhaps very small), so for $T > 0$ the resistance will be greater than zero. Mike Tinkham and his collaborators studied superconductors in $D = 1$ extensively, as discussed in Refs. 1 and 3.

I first worked on this type of problem in the early 1980s with David Abraham, Teun Klapwijk, and Mike Tinkham [4,5]. At the time, there was much interest in the Kosterlitz-Thouless transition, which was proposed as a theoretical description of the phase transition in $D = 2$ neutral superfluids [6]. While it was at first thought that the Kosterlitz-Thouless transition should not occur in superconductors, it was later shown theoretically that it should occur in large but finite samples if the two-dimensional penetration depth were large enough [7]. As described in Section 2, we studied this problem in two-dimensional square lattices of Josephson junctions with zero external magnetic field, and were able to observe a key signature of the transition in our experiments.

The discovery of high-temperature superconductors brought about renewed interest in the Kosterlitz-Thouless transition. Many papers reported a Kosterlitz-Thouless transition in high-temperature superconductors. The transition was sometimes reported in quite thick samples, presumably because the materials are sufficiently anisotropic to decouple the Cu—O planes from each other, making even thick samples effectively two dimensional. Section 3 of this paper describes work done by Max Repaci and other members of my group at Maryland on single-unit-cell films of $YBa_2Cu_3O_{7-\delta}$ in zero magnetic field [8]. These samples are presumably as two dimensional as you can get in a cuprate, yet we showed that they did *not* undergo a Kosterlitz-Thouless transition. The resistance of these unit-cell thick films remained nonzero (but very small) to very low temperatures because their two-dimensional penetration depths are too small, in marked contrast with the Josephson arrays of Section 2.

It seemed natural to next study the $D = 3$ superconducting phase transition in thick high-T_c films. Because of their short coherence lengths, long penetration depths, and high-transition temperatures, fluctuations play a much greater role in high-T_c superconductors than in low-T_c superconductors [9]. Given our work in $D = 2$, the $D = 3$ transition in zero magnetic field seemed like a good place to start. Doug Strachan began this work, but found, surprisingly, that results disagreed with theory. We decided to try our experiment in a magnetic field, because there were a very large number of recent theoretical and experimental papers on the topic, with theory and experiment agreeing that a new type of phase transition (depending on the type of pining in the sample, a vortex-glass [2] or a Bose-glass [10] transition) occurred in a field. Our experimental results in magnetic field were very similar to other people's results. When analyzing our data, however, our conclusions were *not* in agreement with most others': The resistance did not go to zero, suggesting that the superconducting phase transition does not occur in magnetic field for $D = 3$ [11]. These results are discussed in Section 4.

Building on the results obtained in magnetic field, we returned to the $D = 3$ zero field experiments. The results were disturbing: The zero-field results were very similar to the nonzero field results. In particular, the resistance did not appear to be going to zero in the manner expected as temperature was lowered. Matt Sullivan showed [12] that our samples were not sufficiently three dimensional: Even in very thick films (0.32 μm), the experiment's length scales were limited by the film thickness. These results are discussed in Section 5.

2. KOSTERLITZ-THOULESS TRANSITION IN JOSEPHSON-JUNCTION ARRAYS

As discussed briefly in Section 1, thermal fluctuations will cause resistance below T_c in a superconductor if the correlation length, ξ, is larger than the diameter of a superconducting wire. The situation is subtler in two dimensions.

To study superconductivity in $D = 2$, one can make very thin films. While this approach works [13], it can be difficult to make films that are very thin and very uniform.

Another approach is to make square lattices of Josephson junctions. As discussed in Refs. 1, 5, 14, and 15, square Josephson-junction arrays are a discrete version of a two-dimensional superconductor, with the advantage that properties such as the penetration depth can be varied by changing the critical currents of the junctions.

As shown by Kosterlitz and Thouless (KT) [6], the essential fluctuations to consider are vortex–antivortex pairs. Since we are considering zero external magnetic field here, there will be an equal number of vortices and antivortices. KT showed that if the vortex–antivortex interaction is logarithmic in separation, a vortex-unbinding transition occurs. Below a characteristic temperature T_{KT}, each vortex is bound to an antivortex, while above T_{KT}, some vortices will thermally unbind.

In a two-dimensional Josephson-junction array, vortex–antivortex pairs interact via

$$U = \Phi_o i_c \ln\left(\frac{r}{a}\right) \tag{3}$$

where $\Phi_o = h/2e$ is the flux quantum, i_c is the temperature-dependent critical current of one junction, r is the distance between the vortex and the antivortex centers, and a is the lattice spacing of the array [15]. Eq. (3) is true only if

$$r < \lambda_\perp, \tag{4}$$

where $\lambda_\perp$ is the penetration depth for two-dimensional samples, given by

$$\lambda_\perp = \frac{\Phi_o}{2\pi\mu_o i_c}. \tag{5}$$

As long as the sample size is greater than $\lambda_\perp$, Eq. (4) will always be satisfied, and a KT transition can be observed, as first shown in Refs. 7. Note that, by making i_c smaller, $\lambda_\perp$ can be made as large as needed to guarantee that Eq. (4) is true.

Given that the KT transition was predicted to occur, how would one observe it in a Josephson array? Since there are free vortices present for $T > T_{\mathrm{KT}}$, there will be a flux-flow resistance. The prediction for this is

$$R_{\mathrm{HN}} = c_1 R_n\, e^{-\left[\frac{c_2 T_{KT}}{T - T_{KT}}\right]^{1/2}}. \tag{6}$$

Here R_n is the normal-state resistance of the array, and c_1 and c_2 are constants of order unity [7,16]. (The formula for arrays is more complicated, but this thin-film Halperin-Nelson version is sufficient for the discussion here. See Ref. 15.) There are subtle problems with the formula, however: It is only correct for small currents, and for temperatures close to T_{KT}, though the definitions of "close to" and "small" are nontrivial. Nevertheless, as other people measuring thin films and arrays before us had found [17], Eq. (6) seemed to agree with the data. This is shown in Fig. 2.

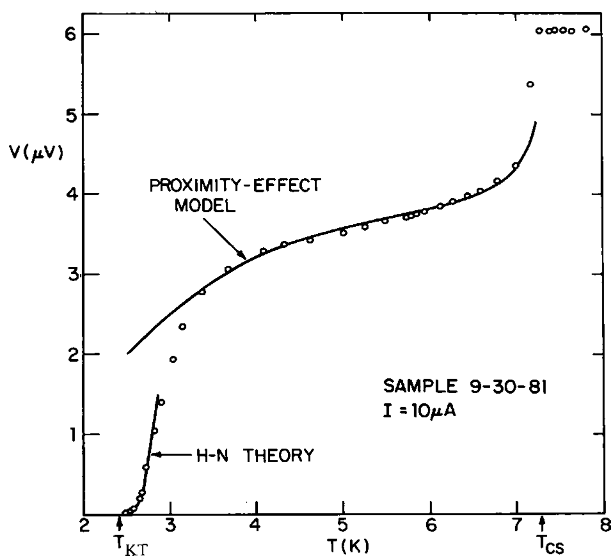

Fig. 2. Voltage vs. temperature at a constant 10 μA current for a 1000 $\times$ 1000 Josephson-junction array. Data are open circles, and solid labeled H-N (Halperin-Nelson) theory is a fit to the complete form of Eq. 6. From Ref. 4.

When we arrived at this point in the research, Mike Tinkham made a typical Tinkham comment. He told us that it is easy to fit data to reasonable three-parameter functions, and that Eq. (6) was reasonable: It predicted that the resistance should drop as the temperature was lowered. He reminded us that a single overdamped junction with small critical current would have a measurable resistance due to thermal fluctuations that also dropped rapidly as the temperature was lowered. In the limit of small current, the prediction for this is [18]

$$R_{\mathrm{AH}} = R_{\mathrm{n}}\left[I_{\mathrm{o}}\left(\frac{\Phi_{\mathrm{o}} i_{\mathrm{c}}}{\pi kT}\right)\right]^{-2} \approx R_{\mathrm{n}}\frac{2\Phi_{\mathrm{o}} i_{\mathrm{c}}}{kT}\, e^{-\frac{2\Phi_{\mathrm{o}} i_{\mathrm{c}}}{\pi kT}}. \quad (7)$$

Here I_{o} is the zeroth-order-modified Bessel function, and the approximate version holds when the argument of the Bessel function is much greater than one.

While Eq. (7) is nonzero for $T > 0$, it does predict a rapid drop in resistance as the temperature is lowered. At first sight, in fact, it was hard to prove that Eq. (7) did not explain the data, especially since i_{c} depended on T in a manner that was not precisely known from experiment.

Fortunately, Ref. 16 provided a more demanding way to test the KT theory. For $T \leq T_{\mathrm{KT}}$, they showed that the voltage should be a power of the current, of the form

$$V \propto I^{a(T)}. \quad (8)$$

where Eq. (8) is also restricted to small currents. A striking prediction of the Halperin-Nelson theory is that just below T_{KT},

$$a(T_{\mathrm{KT}-}) = 3. \quad (9)$$

Eq. (6) predicts ohmic behavior just above T_{KT}, so that V is proportional to I, or

$$a(T_{\mathrm{KT}+}) = 1. \quad (10)$$

The KT transition should thus cause a *discontinuous* change in the power-law behavior at T_{KT}. By contrast, Eq. (7) predicts ohmic behavior for *all* temperatures.

The way to see this behavior is to plot current–voltage (IV) curves on a log–log plot, as is done in Fig. 3. Since $\log(V) = a(T)\log(I) + \text{constant}$, the slope of the log–log IV curves should change from 3 to 1 at T_{KT}. The result of this is shown in Fig. 4, where a slightly broadened jump from 3 to 1 can be seen around $T = 2.4$ K. Note that Eq. (8) not only implies zero resistance, but also zero critical current, for $T < T_{\mathrm{KT}}$.

As further proof that Eq. (7) does not account for the data, Ref. 5 shows that $R_{\mathrm{AH}}(T_{\mathrm{KT}}) = 0.82 R_{\mathrm{N}}$, while, of course, $R_{\mathrm{HN}}(T_{\mathrm{KT}}) = 0$. The evidence for a KT transition in our arrays was thus very strong, indicating that these two-dimensional samples are indeed superconducting.

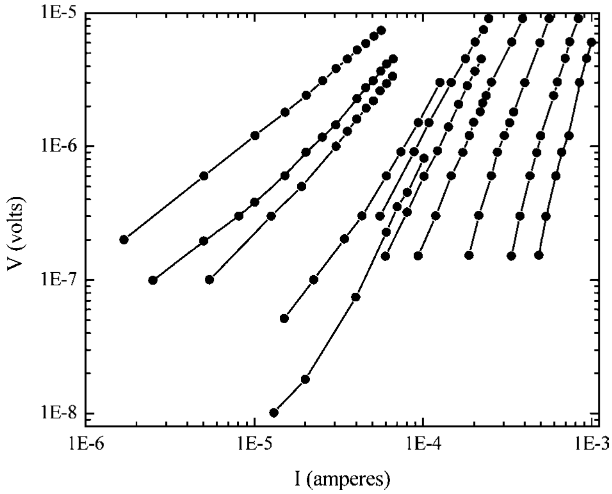

Fig. 3. Log voltage vs. log current for a square Josephson-junction array. These data represent a small subset of the actual data, and were generated by reading points off of continuous IV curves originally made on an XY plotter. Curve at upper left has $T = 2.75$ K, right most curve has $T = 1.95$ K.

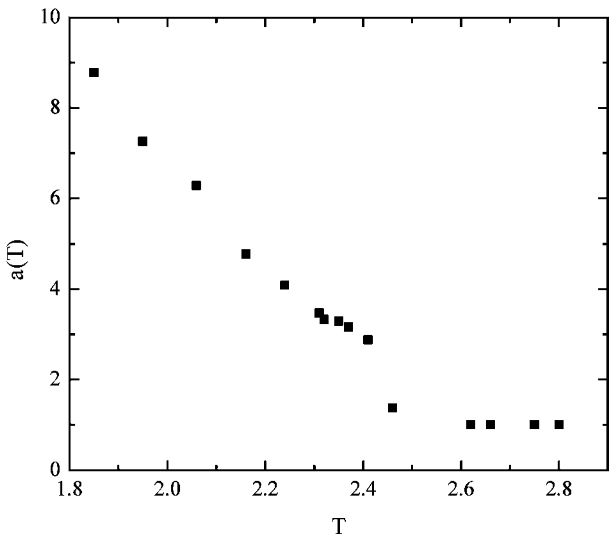

Fig. 4. $a(T)$ vs. T taken from the slopes of $\log I$–$\log V$ plots for voltages in the 10-nV range. From data in Ref. 4.

3. ABSENCE OF A KOSTERLITZ-THOULESS TRANSITION IN CUPRATE SUPERCONDUCTORS

At the University of Maryland, Max Repaci made and measured single-unit-cell-thick films of $YBa_2Cu_3O_{7-\delta}$ to see if a Kosterlitz-Thouless transition occurred [8]. In such films, vortex–antivortex pairs interact via a potential

$$U = 2\pi n_s^* \frac{\hbar^2}{m^*} \ln\left(\frac{r}{\xi}\right) \tag{11}$$

where n_s^* is the number of Cooper pairs per unit area [5]. Note that Eq. (11) is of the same form as Eq. (3). Like Eq. (3), Eq. 11 is true only if $r < \lambda_\perp$, where in a superconductor of thickness d,

$$\lambda_\perp = \frac{\lambda^2}{d}, \tag{12}$$

where λ is the bulk penetration depth for the material [18].

The passage of a decade allowed us the significant advantage of easily acquiring data with a computer. *IV* curves for a typical sample are shown in Fig. 5. (The advantages of Fig. 5 over Fig. 3 are very great indeed!)

Figure 5 certainly appears to show a superconducting transition. At the highest temperatures, the *IV* curves are ohmic, with slope 1 on a log–log plot. At the lowest temperatures, the voltage drops very rapidly as a function of current, with no indication of ohmic behavior. At intermediate temperatures, the voltage drops rapidly as a function of current at high currents, and is ohmic at low currents. The low-current ohmic "tails" are no longer visible for temperatures below about 22 K on the plot.

The question is, do the ohmic tails disappear because the sample becomes superconducting (presumably by undergoing a KT transition), or do they continue to occur at voltages below the resolution of the voltmeter? A good way to explore this issue is to plot $d\log(V)/d\log(I)$ vs. $\log(I)$, as is done in Fig. 6. From Eqs. (6) and (8), we expect that $d\log(V)/d\log(I) = 1$ for $T > T_{KT}$, $d\log(V)/d\log(I) = 3$ for $T = T_{KT}$, and $d\log(V)/d\log(I) = a(T)$, with $a(T) > 3$, for $T < T_{KT}$, all at low currents.

Rather than showing a KT transition, Fig. (6) indicates that there is not a phase transition at all. At the highest temperatures, $d\log(V)/d\log(I) = 1$ for all currents, indicating the normal state. As the temperature is lowered, $d\log(V)/d\log(I) > 1$ at intermediate currents, but bends back down toward 1 at low currents. As the temperature is lowered further, the trend down is still evident, but $d\log(V)/d\log(I)$ does not reach 1 because of limited voltmeter sensitivity. Furthermore, it is clear that there is no isotherm where $d\log(V)/d\log(I) = 3$ or any other larger constant over any appreciable range of current.

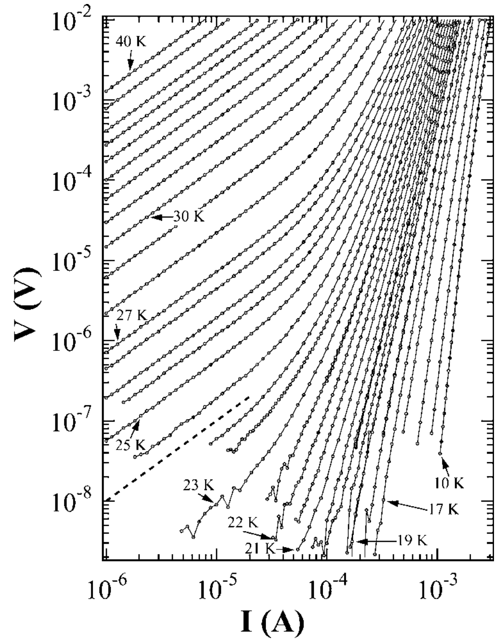

Fig. 5. Current–voltage characteristics plotted on a log–log scale for a unit-cell thick $YBa_2Cu_3O_{7-\delta}$ film in zero magnetic field. Lines connecting the points are guides for the eye. The dashed line near the bottom of the plot has a slope of 1, representing ohmic behavior. Temperatures range from 40 K at the top to 10 K at the bottom. From Ref. 8.

The simplest explanation for Fig. (6) is that there is not a KT transition, perhaps because $\lambda_\perp$ is smaller than the sample size. As discussed in Ref. 8, the data imply that $\lambda_\perp(T = 0) \approx 160\ \mu$m. Equation (12), combined with single-crystal measurements of $\lambda(0)$, imply a value of $\lambda_\perp(T = 0)$ which is a factor of 4 smaller than the estimate in Ref. 8, as might be expected given that single crystals are cleaner and have much higher transition temperatures than unit-cell films. Both estimates are smaller than the 200 μm sample width, so that Eq. (11) is not valid for all vortex–antivortex pairs in the sample. Indeed, when $r > \lambda_\perp$, the interaction energy approaches a constant, which guarantees that more widely separated vortex–antivortex pairs will be thermally unbound at *all* nonzero temperatures [8,15]. The absence of a KT transition, and the occurrence of nonzero resistance at low temperatures, are thus in agreement with theory.

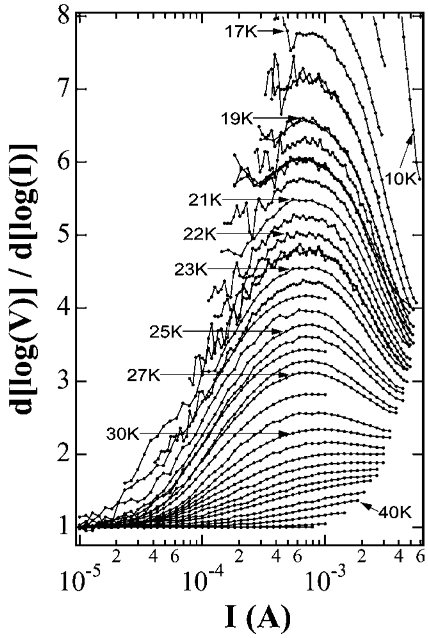

Fig. 6. $d\log(V)/d\log(I)$ as a function of current. Temperatures range from 10 K at the upper right to 60 K at the bottom. Lines connecting the points are guides for the eye. From Ref. 8.

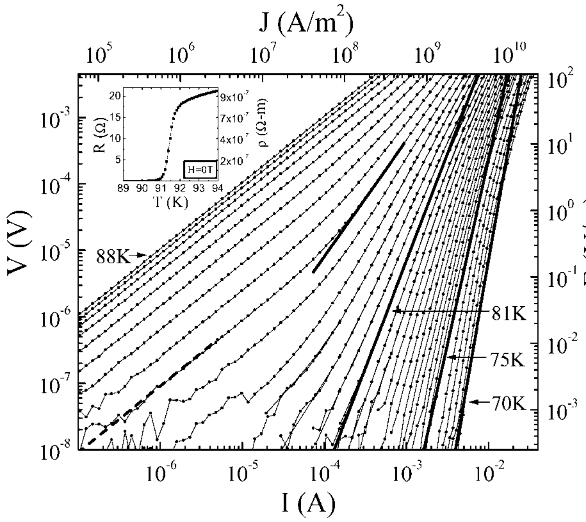

Fig. 7. Current–voltage characteristics plotted on a log–log scale for a 220-nm-thick $YBa_2Cu_3O_{7-\delta}$ film in 4 tesla magnetic field. The dashed line at the lower left has slope 1, while the solid lines at 81, 75, and 70 K are fits to simple power laws. The inset is $R(T)$ vs. T in ambient field. From Ref. 11.

Note that Eq. (12) implies that thicker samples are even less likely to undergo a KT transition. We concluded that $YBa_2Cu_3O_{7-\delta}$ does not undergo a KT transition—earlier work failed to distinguish between very small resistance and zero resistance.

4. THREE-DIMENSIONAL SUPERCONDUCTORS IN A MAGNETIC FIELD

Starting with the pioneering theoretical work of Fisher [19] and experimental work of Koch *et al.* [20], a new picture of the transition for bulk type-II superconductors in a magnetic field emerged in the early 1990s [21]. On the basis of work done on conventional superconductors, it had been believed that a magnetic field led to nonzero (although possibly extremely small) resistance [1]. The new consensus was that a transition to a true zero-resistivity state occurs in the presence of a magnetic field. Various theories have been proposed for this phase transition, including a vortex-glass transition [2,19], which is predicted to occur when disorder in the superconductor is uncorrelated, and a Bose-glass transition [10], which is predicted to occur in the presence of correlated disorder. While these theories apply to different situations, both predict that the resistivity should be zero below a transition temperature.

Doug Strachan at Maryland decided to take another look at this problem [11,21], based on difficulties in understanding our measurements of bulk samples in zero field. Figure 7 shows *IV* curves from a typical high-quality $YBa_2Cu_3O_{7-\delta}$ thick film. (This film was laser ablated onto a $SrTiO_3$ substrate and had a thickness of 220 nm.) The pinning in such samples is uncorrelated, so theory predicted, and many earlier experiments seemed to confirm, that a vortex-glass transition should occur.

Qualitatively, Fig. 7 is very similar to Fig. 5. We used the same approach to examine the data more carefully, by plotting a $d\log(V)/d\log(I)$ vs. $\log(I)$ plot, shown in Fig. 8. This graph is very similar to Fig. 6.

It is important, however, to remember that this is a three-dimensional sample, while the data shown in Figs. 5 and 6 come from a two-dimensional sample, so one cannot draw conclusions about the presence or absence of a phase transition based on Eqs. (8)–(10). Fortunately, scaling [2,21] leads to testable predictions for the behavior of *IV* curves.

The basic prediction is that

$$\frac{V}{I} = \xi^{D-2-z}\chi_{\pm}\left(\frac{I\xi^{D-1}}{T}\right), \qquad (13)$$

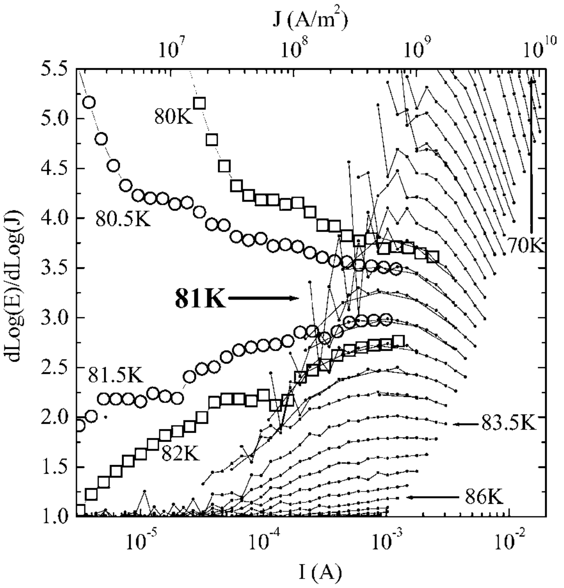

Fig. 8. Small solid circles are $d\log(V)/d\log(I)$ as a function of current. The open symbols are extrapolated data based on scaling showing a change in slope from positive to negative as temperature is lowered—a signature of the phase transition that is not present in the actual data. From Ref. 11.

where ξ and z are defined in Eqs. (1) and (2), and $\chi_{\pm}$ are two unknown functions, one (+) for above T_c, the other (−) below T_c.

Eq. (13) has two useful limiting forms. For small currents and $T \geq T_c$, it can be shown that

$$\frac{V}{I} \sim \xi^{D-2-z} \sim \varepsilon^{\nu(2+z-D)}, \tag{14}$$

where ε is defined below Eq. (1). We thus see that samples should be ohmic for small currents above T_c. Note that if $D = 2$ and $z = 2$, this reproduces Eq. (6) when the two-dimensional equation for ξ is used, see Refs. 5,7, and 16.

The second useful limiting form applies for $T = T_c$ (and only at $T = T_c$), where Eq. (13) implies that

$$V \sim I^{(z+1)/(D-1)}. \tag{15}$$

Note that, for $z = 2$ and $D = 2$, Eq. (15) agrees with the KT result, Eq. (8).

Standard scaling analysis assumes that a transition does occur. Assuming this to be correct for the time being, Eq. (15) predicts that the IV curve at $T = T_g$ should be a straight line on a log–log plot, with slope given by $(z+1)/(D-1)$. (I use T_g here in place of T_c to indicate that the measurements are in field.) The dark solid line drawn in Fig. 7 is a power-law fit to the IV curve at 81 K which looks closest to a power law (i.e., it looks straight on a log–log plot.). Using $D = 3$ and Eq. (15), this determines a value of $z = 5.46$.

Following the standard analysis, we next use $T_g = 81$ K, $z = 5.46$, and Eq. (14) to determine ν. The resistances R_L are read off of the low-current tails in Fig. 7, and plotted on a log–log plot, as shown in the inset to Fig. 9a. It is seen that below about 87 K, a

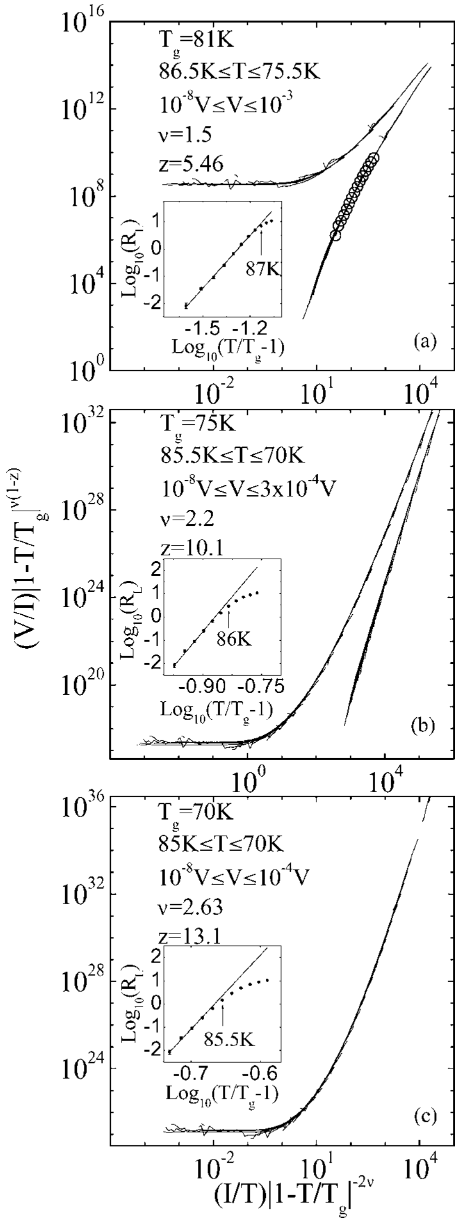

Fig. 9. Collapses of the data from Fig. 7 using Eq. (16), using different values for T_g. All of the data collapses look good, which demonstrates that the technique can be too flexible if used uncritically. From Ref. 11.

good fit is obtained, with deviations at higher temperatures. Equation (14) yields a value $\nu = 1.5$, again consistent with other values in the literature.

Equation (13) can be rewritten as

$$\frac{V}{I}\xi^{2+z-D} = \chi_{\pm}\left(\frac{I\xi^{D-1}}{T}\right). \tag{16}$$

Equation (16) predicts that a plot of $V\xi^{2+z-D}/I$ against $I\xi^{D-1}/T$ should "collapse" all of the data in the critical regime onto one of two curves, χ_+ for $T \geq T_g$ and χ_- for $T \leq T_g$. This data collapse is shown in Fig. 9a.

The data collapse shown in Fig. 9a is very impressive to the eye. The apparent success of the data collapse is widely taken to indicate the data scale. This, in turn, would indicate that a phase transition has taken place.

Unfortunately, Fig. 8 and Eq. (15) taken together indicate that a phase transition has not taken place. No nonnormal state data in Fig. 8 fall on a straight horizontal line, as predicted by Eq. (15).

The problem is that the scaling approach must be applied with more caution. Note that qualitatively, at least, *all* the isotherms with $T \leq 81$ K appear to be straight over some range in V in Fig. 7. (Note, however, that Fig. 8 indicates that this is not true.) They would thus all *appear* to satisfy Eq. (15), which suggests that T_g may not be uniquely determined by the standard procedure. To test this idea, we did the standard scaling analysis with a different value of $T_g =$ 75 K. The result of this scaling analysis is graphed in Fig. 9b. Remarkably, this data collapse also looks very good.

Taking this to the extreme case, Fig. 9c shows the result of choosing $T_g = 70$ K, the lowest temperature measured in the experiment. Here, since all the data are from temperatures above the nominal T_g, all the data collapse onto only one curve, corresponding to χ_+ in Eq. (9). Once again, the collapse appears to be quite good.

The problem with the conventional scaling collapse approach is that it is too flexible, as Fig. 9 shows. A more stringent test of scaling is to use Eq. (13) to generate predictions based on extrapolation of the actual data, as is done in the open symbols in Fig. 8. These extrapolations show a clear signature of the phase transition which is not present in the actual data: All curves tend toward ohmic behavior at low currents for $T > T_g$, while all curves diverge for $T < T_g$, with a break at at the critical isotherm. On the basis of this analysis, we concluded that the data are inconsistent with the occurrence of a superconducting phase transition.

5. THREE-DIMENSIONAL SUPERCONDUCTORS IN ZERO MAGNETIC FIELD

While the data discussed in the previous section strongly indicated that the resistance became small but not zero at lower temperatures, they did not tell us why there was no superconducting phase transition. Were the old theories, which predicted small but nonzero resistance in a magnetic field, correct after all? Or was something else going on?

It seemed like a good time to return to the $D = 3$ zero magnetic field experiment, which Matt Sullivan did [12]. The existence of a superconducting phase transition in this case is not in doubt, and furthermore, there are theoretical estimates for the critical exponents ν and z. Very close to $T_c(|T - T_c|$ smaller than about 2 K [9]), the transition is expected to be of the three-dimensional XY type, with $\nu \cong 0.67$ and $z = 2$ for diffusive dynamics [2]. Interestingly, however, researchers have found vortex-glass like exponents $\nu \cong 1.1$ and $z \cong 8.3$ in small fields (<10 mT) [22] while others find three-dimensional XY exponents when extrapolating to zero field from higher fields [23] and in crystals [24].

Figure 10 is a log(V)–log(I) plot for a 210-nm-thick $YBa_2Cu_3O_{7-\delta}$ film in zero magnetic field

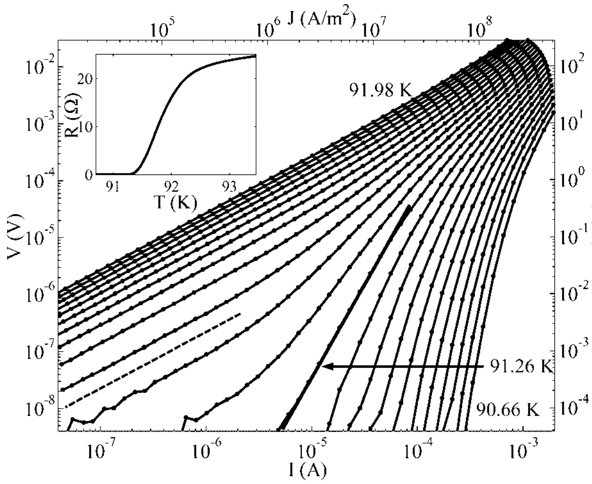

Fig. 10. Current–voltage characteristics for a 210-nm $YBa_2Cu_3O_{7-\delta}$ film in zero magnetic field. Curves are separated by 60 mK. The dashed line indicates a slope of 1, or ohmic behavior. Inset shows $R(T)$ at 10 μA. From Ref. 12.

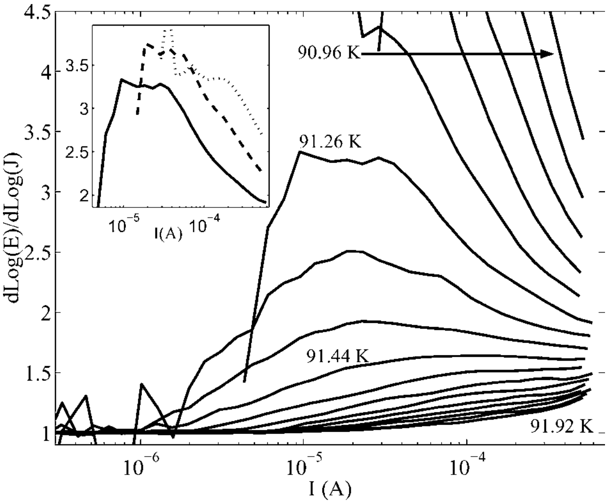

Fig. 11. $d\log(E)/d\log(J)$ vs. I for the IV curves of Fig. 10. The inset shows the 91.26 K isotherm for three-bridges widths on the same film, 20 μm (solid line), 50 μm (dashed line), and 100 μm (dotted line), which do not agree as a function of I. From Ref. 12.

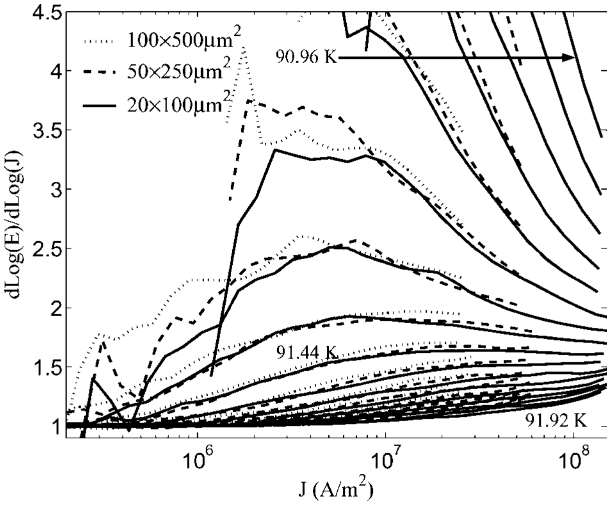

Fig. 12. $d\log(E)/d\log(J)$ vs. J for three different-width samples fabricated from the same film. The crossover from nonohmic to ohmic behavior clearly depends on J. From Ref. 12.

[12]. Once again, the data are qualitatively consistent with a transition occurring, with ohmic low-current tails being visible for $T > 91.26$ K, and not being visible at lower temperatures.

Figure 11 is a derivative plot for the data in Fig. 10. It is qualitatively similar to Eqs. (6) and (8). In particular, if a phase transition were present, there would be one curve, at $T = T_c$, that is straight and horizontal, separating curves at higher temperature with positive derivative from those at lower temperature with negative derivative. As with the other derivative plots in this paper, this is not what is seen here: A simpler explanation of the data is that all of the underlying curves are the same, with the only differences being due to voltmeter resolution.

Earlier work had pointed out the possibility that in thin films, fluctuation dynamics can cross over from three-dimensional to two-dimensional behavior [2,25,26]. The idea is that a current density J probes fluctuations of a typical size L_J given by [2]

$$L_J = \left(\frac{ckT}{\Phi_o J}\right)^{1/2} \tag{17}$$

where c is a constant of order the $YBa_2Cu_3O_{7-\delta}$ film anisotropy parameter, about 0.2. As long as $L_J < d$, the film thickness, the measurements will be probing three-dimensional fluctuations. Once $L_J > d$, however, the measurements will be probing two-dimensional fluctuations.

This crossover from three dimensions to two dimensions provides a qualitative explanation for the behavior seen in Fig. 11, and can also be checked quantitatively. The inset to Fig. 12 shows derivative plots taken on samples with three different widths made from the same film and measured at the same temperature, and it is seen that the curves do not lie on top of each other. (They would fall on top of each other if the effect depended on current instead of current density.) By contrast, Fig. 13 plots data

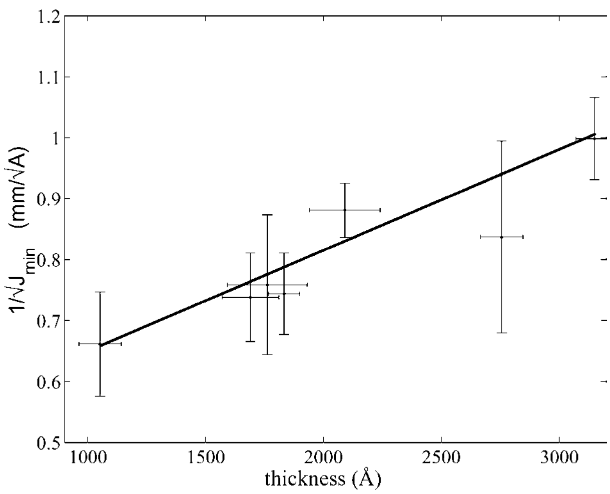

Fig. 13. $1/\sqrt{J_{\min}}$ vs. d for eight different thickness films. The straight line fit indicates quantitatve agreement with Eq. (18). From Ref. 12.

from the same three samples using current density J as a variable, rather than current I. Within experimental uncertainty, all three curves at any given temperature lie on top of each other, in agreement with Eq. (17).

To further test Eq. (17), Matt Sullivan made films with thicknesses varying between 95 nm and 320 nm. Using a method described in Ref. 12, he inferred a value of the current, $J_{\min}$, at which deviations from three-dimensional behavior first occur. If this deviation occurs when $L_J = d$, then Eq. (17) predicts that

$$J_{\min} = \frac{ckT}{\Phi_o d^2}. \qquad (18)$$

Figure 13 shows that this is indeed the case. Quite remarkably, the IV curves are dominated by finite-thickness effects, even in quite thick films.

6. SUMMARY AND CONCLUSIONS

I have discussed a number of different systems—two-dimensional Josephson-junction arrays, two-dimensional unit-cell $YBa_2Cu_3O_{7-\delta}$ films, and three-dimensional thick (fractions of a micrometer) $YBa_2Cu_3O_{7-\delta}$ films. In two dimensions, whether or not the KT transition occurs depends on the width W of the sample relative to $\lambda_\perp$. In the arrays, $W < \lambda_\perp$, and the signature of the KT transition, a change from cubic to linear IV curves at T_{KT}, can be seen. In unit-cell $YBa_2Cu_3O_{7-\delta}$ films, $W > \lambda_\perp$, and the transition is not seen.

In three-dimensional samples, scaling provides a powerful tool for determining whether or not samples are truly superconducting. While a "by eye" data collapse such as those shown in Fig. 9 can lead to the mistaken conclusion that a transition has occurred, when the technique is used carefully it is very powerful. Our in-field data were not consistent with a transition to a state of zero resistance, which led us to suggest that a vortex-glass transition may not occur. Quite alarmingly, our zero-field data led us to a similar conclusion. In the latter case, however, we believe that ubiquitous finite-size effects interrupt the transition, even in our thickest films.

An important question is whether finite-thickness effects are causing the samples to not become superconducting in field, or something else is happening. This question is currently being pursued at Maryland. In addition, we are making measurements on single crystals, which are very much thicker than films. Our initial results indicate that finite size effects are not visible in the crystals, but more work is needed. Finally, we are measuring other properties, such as microwave response and heat capacity, which are independent ways of studying whether or not samples are really superconducting.

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Mike Tinkham taught me a lot when I worked with him. There is no doubt in my mind that the work we did at Harvard was very good: His role in that work means that I am not being conceited by saying so. I hope that my subsequent work occasionally measures up to his high standards. I have had many other collaborators in the work described here, as well as in related work that could not be discussed due to space constraints; all of them have been a source of knowledge and inspiration. I especially acknowledge the primary authors of the work described here, David Abraham, Max Repaci, Doug Strachan, and Matt Sullivan. I had a lot of fun working with all of you.

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